

# A MATHEMATICAL THEORY OF ÆSTHETICS<sup>1</sup>

## I

### RELATION OF THE THEORY TO EARLIER ÆSTHETIC THEORIES

#### 1. INTRODUCTION

**I**N a paper presented before the International Mathematical Congress at Bologna in 1928<sup>2</sup> I proposed a mathematical theory of æsthetics as follows:

“The typical æsthetic experience can be regarded as containing three successive phases: (1) a preliminary effort, necessary to perceive the object and proportional to its complexity  $C$ ; (2) the feeling of pleasure or æsthetic measure  $M$  which rewards this preliminary effort; and finally (3) a realization that the object embodies a certain harmony, symmetry, or order  $O$ , more or less concealed, which seems to be a necessary condition, if not sufficient, for the æsthetic experience.

“Thus there arises almost at once the question of determining in a given case to what extent this æsthetic measure is simply the effect of the density of the relations of order,

<sup>1</sup>A series of three public lectures, under the auspices of the Rice Institute Lectureship in Music, delivered in the Physics Amphitheatre, January 4, 5 and 6, 1932, by George David Birkhoff, Ph.D. (Chicago), Sc.D. (Wisconsin and Brown), Professor of Mathematics in Harvard University.

<sup>2</sup>“Quelques Éléments Mathématiques de l'Art,” *Atti del Congresso Internazionale dei Matematici*, Bologna (1929), Vol. 1.

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compared with the complexity. And so it seems natural to propose such a formula as

$$M = \frac{O}{C},$$

At the same time I gave preliminary indications of the application of this æsthetic formula to polygonal forms,<sup>1</sup> tiles and vases.

The program announced then has been followed up systematically. What I wish to lay before you in these lectures which I have the great honor to give at the Rice Institute, is an account of this theory in its relation to earlier æsthetic theories, and its applications to poetry and music.

I realize how daring an attempt to found a mathematical theory of æsthetics must appear, but I hope that you will agree with me at the conclusion of this first lecture that the attempt is historically justified.

Before entering upon this historical treatment, it is desirable to state in more precise terms the point of view of the mathematical theory.<sup>2</sup>

The branch of knowledge called Æsthetics deals with æsthetic feeling and with the æsthetic objects which produce it. These objects fall into classes such that the objects of a definite class may be compared with respect to æsthetic value, while those of different classes cannot be thus compared. In this way arises the fundamental problem of Analytic Æsthetics, namely, for each class to determine as far as possible the æsthetic factors involved, and their relative importance.

If we accept the mathematical theory indicated above, the

<sup>1</sup>A more complete development of the theory of polygonal forms along these lines will be found in an article "Polygonal Forms," Sixth Yearbook of the National Council of Teachers of Mathematics, New York, 1931.

<sup>2</sup>See in this connection an article, "A Mathematical Approach to Æsthetics," *Scientia*, 1931.

following formulation of the fundamental æsthetic problem may be made: *within each class of æsthetic objects to define the order  $O$  and the complexity  $C$  so that the ratio  $M = O/C$  yields the æsthetic measure of the different objects of the class.*

A psychological justification of such a formula may be given. The act of perception of an æsthetic object involves certain automatic motor adjustments, and the feeling of effort or tension which always accompanies perception appears as a summational effect due to the tensions which accompany the various automatic adjustments. Thus if  $A, B, C, \dots$  represent these adjustments, each with its index of tension  $a, b, c, \dots$ , and if these respective adjustments are made  $r, s, t, \dots$  times, we may consider the sum

$$C = ra + sb + tc + \dots$$

to represent the complexity.

On the other hand, the order  $O$  corresponds to certain associations called into play by the act of perception. An instance of such an association would be that of symmetry. More precisely, if associations of various types  $L, M, N, \dots$  take place, each with its index of tone of feeling,  $l, m, n, \dots$  and if these occur  $u, v, w, \dots$  times, then we may regard the total tone of feeling (positive or negative) to be

$$O = ul + vm + wn + \dots$$

representing the order  $O$  of the object.

According to our theory it is the intuitive estimate of the amount of order,  $O$ , inherent in the æsthetic object, as compared with its complexity,  $C$ , from which arises the feeling of the æsthetic measure,  $M$ , of the object considered; the fundamental formula embodies this conjecture.

An instructive analogy is the following. Among business enterprises those are regarded as most successful in which

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the annual profit  $p$  is largest in comparison to the investment  $i$ ; thus the ratio  $p/i$  is regarded as the economic measure of success. Now in the æsthetic experience, the effort of attention measured by  $C$  corresponds to the investment, and the order  $O$  corresponds to the profit. By analogy it is the ratio  $O/C$  which represents the æsthetic measure  $M$ .

It is obvious that any such mathematical theory can only represent the facts for certain simple classes of æsthetic objects in which the æsthetic factors involved are essentially mathematical or formal. These formal factors comprehend symmetry, balance, repetition, sequence, etc. On the other hand there are connotative factors which are obviously beyond the reach of any such theory. An instance would be the associations involved in the meaning of a beautiful poem.

Thus the quantitative application of the mathematical theory will be limited to that part of the æsthetic effect which is due to the formal elements of order of simple type. However, the formula can be held to be qualitatively applicable to the complete æsthetic experience.

With these general facts concerning the mathematical theory in mind, we propose first of all to pass in brief review various earlier æsthetic theories. The account given can only be one of broadest outline, of course, for the literature involved is extremely extensive. Our main interest will be to interpret the principal advances in terms of the mathematical theory, and to observe how far earlier writers, beginning with Plato and Aristotle, have perceived the presence of mathematical elements in art, and what rôle they have ascribed to these elements.

The pleasurability of art as based upon its sensuous nature, its usefulness for purposes of instruction, its mystic quality due to the presence of connotative or occult formal

elements of order; all these are obviously important aspects, each of which has from time to time been looked upon as of dominant importance. Thus have arisen *hedonistic*, *pedagogic*, and *mystical* theories of art and æsthetics.

From the scientific point of view, however, it seems almost meaningless to declare that one of these aspects is the most fundamental.

In contrast with hedonistic, moralistic, and mystical theories may be placed those *analytic* theories which attempt to discover the specific æsthetic factors involved in the several fields of art, to appraise the rôles of these factors, and then to formulate general laws so far as possible. Such theories are obviously concerned with what we have called the fundamental problem of æsthetics. Our own mathematical theory finds its place among these analytic theories, but is distinguished from the others in that it aspires to provide a quantitative solution of the fundamental problem, at least as far as the formal side of art is concerned.

It seems almost obvious that æsthetics, if it is to be scientific, must be approached from the analytic point of view and must concern itself chiefly with the formal aspects of art.

For this reason we shall allude only in the briefest terms to those æsthetic writings which are not analytic in character, despite the fact that some of them are literary works of art of a very high order.

## 2. PLATO

From very early times there are to be found numerous critical reflections concerning poetry, painting, and sculpture. In fact it is inherent in the nature of the æsthetic process that objects of the same kind provoke comparison

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and that this comparison leads to increased understanding of the underlying æsthetic factors.

Now it was obvious first of all that painting and sculpture are closely akin in that they are representative or mimetic, and that poetry shares in this attribute because of its capacity to suggest a series of visual and auditory images. On the other hand poetry was seen to differ from painting and sculpture because of its much greater pedagogic power. In this respect poetry is similar to philosophy, but differs from it in that poetry conveys ideas in an indirect pleasing manner rather than in the direct neutral manner characteristic of philosophical discourse.

From his philosophic point of view Plato was led to assign an inferior position to art because of this mimetic quality: for a work of art was the imitation of an object, and these in turn were but faint copies of the fundamental Platonic Ideas; thus art, as the imitation of an imitation, could scarcely merit serious philosophic consideration. Furthermore the admixture of the sensuous element in art did not meet with his approval. In consequence of these considerations Plato proposed to exclude poets from his ideal Republic.

Such a criticism of art is evidently unsound since it ignores the fact that art is imaginative and expressive.

Plato was also interested in the problem of the beautiful as Socrates and other philosophers had been before him. In general it may be said that for him the beautiful was not that found in art, but rather that of objects in nature. Thus in the *Hippias maior* a beautiful maiden, mare, lyre, and vase are instanced. The beauty of laws and of actions is also remarked upon.

In this dialogue several definitions of the beautiful are in turn examined and rejected: that which is fitting, or useful,

or good; that which delights the sight and hearing. It is evident that the first definitions are negative and therefore incomplete; they refer to those conditions which must be satisfied before beauty can be possible. The second definition is also finally rejected because no common element in the senses of sight and of hearing is found, so that beauty would appear to be two things, instead of one.

But there is an underlying common element. Space is what mathematicians call a "three-dimensional metric manifold"; likewise time is a one-dimensional metric manifold. Spatial form and temporal form are therefore of the same abstract nature, and the æsthetic enjoyment of spatial and temporal objects arises in large measure from the formal relations of metric manifolds. Goethe's characterization of architecture as frozen music embodies this truth suggestively.

Plato himself recognizes the importance of this mathematical element, for we read in the *Philebus*: "If arithmetic, mensuration, and weighing be taken out of any art that which remains will not be much," and again, "For measure and proportion always pass into beauty and excellence." In connection with this second passage it is made explicitly clear that he is referring to beauty of geometric form, as exemplified for instance by a circle, and beauty of musical form, as exemplified by a pure musical note.

### 3. ARISTOTLE

Aristotle, inventor of formal or syllogistic logic, author of the *Poetica* and the *Rhetorica*, saw more clearly than Plato that art is expressive and not merely imitative. At the same time it was not this characteristic which seemed to him fundamental, but rather the characteristic of mathematical form: "those are mistaken who affirm that the mathematical

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sciences say nothing of beauty or goodness. For if they most especially discern and demonstrate the facts and definitions relating to them, though without naming the qualities in question, that is not keeping silence about them. The main elements of beauty are order, symmetry, definite limitation, and these are the chief properties that the mathematical sciences draw attention to.”<sup>1</sup>

In his very searching analysis of poetry, Aristotle deals mainly with æsthetic factors other than that of the musical quality which we shall consider. His concept of tragedy as “a representation of an action noble and complete in itself, and of appreciable magnitude, in language of special fascination, using different kinds of utterance in the different parts, given through performers and not by means of narration and producing by pity and fear, the alleviating discharge of emotions of that nature”<sup>2</sup> recognizes this musical factor, for by “language of special fascination” is certainly meant that involving rhythm and melody.

### 4. PLOTINUS

The late Greek philosopher Plotinus is the first great representative of the mystical point of view referred to above. He refused to admit that beauty is identifiable with mere symmetry: “beauty is rather a light that plays over the symmetry of things than the symmetry itself, and in this consists its charm. For why . . . are the more lifelike statues the more beautiful, though the others be more symmetrical . . . except that this living beauty is more desirable?”<sup>3</sup> Here “light” refers to actual light in the physical sense. According to him things are beautiful because they

<sup>1</sup> *Metaphysics*.

<sup>2</sup> *Poetics*. See Bosanquet, *History of Æsthetics*, p. 64.

<sup>3</sup> *Ennead*.



participate in having the attribute of rational form, characteristic of the soul, rather than because of symmetry.

The reaction of Plotinus against the point of view of Plato and Aristotle is justified to the extent that it stresses the inadequacy of any purely formal analysis of a work of art. But when he proceeds to interpret the æsthetic experience in a purely mystical manner, it does not appear that any real advance is accomplished.

## 5. THE GREEK VIEW

It appears then that the prevailing Greek view emphasized the importance of the formal elements in art. Bosanquet says:<sup>1</sup> "the one true æsthetic principle recognized by Hellenic antiquity in general" is that "beauty consists in the imaginative or sensuous expression of unity in variety . . . The relation of whole to part—a slightly more concrete expression for unity in variety—has never been more perfectly elucidated or more justly appreciated than by Plato and Aristotle . . . Moreover the relation of the one to the many or of the part to the whole is represented in comparative purity by geometric figures, or again by rhythmic or spatial intervals that bear numerical relation to one another. And for this reason Greek philosophy is inclined to select form, ratio, or proportion as the pure and typical embodiment of beauty."

## 6. LUCA PACIOLI. MICHELANGELO

Although it was not until the seventeenth century that important new æsthetic ideas appeared, it is interesting to observe that, even earlier, mathematicians and artists were led to ascribe peculiar æsthetic merit to certain numerical proportions. Thus the mathematician Luca Pacioli in his

<sup>1</sup> *History of Aesthetics*.

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*De divina proportione* of 1509 considers the "golden section" of a linear segment to be fundamentally important in this way. This section divides the segment so that the shorter part is to the longer as the longer is to the whole segment.

Michelangelo, close friend of Pacioli, ascribed certain simple proportions to the ideal human figure.

The mathematical theory which is here adopted provides no justification for any mystic Pythagorean dogma in the field of æsthetics, although it recognizes the importance of numerical relationships in certain cases.<sup>1</sup>

### 7. FRACASTORO

What appears to be the first explicit statement that beauty must always be relative to objects of a definite class was made by the physician and poet Fracastoro in his *De poetica*<sup>2</sup> of 1555.

This truth is of course involved in our formulation of the fundamental æsthetic problem. It is more or less implicit in the works of Plato and Aristotle.

### 8. WIT AND TASTE

In the seventeenth and early eighteenth centuries a great deal of suggestive æsthetic discussion turns upon the discussion of "wit" and "taste."<sup>3</sup> It is difficult, however, to attribute the origin of these notions with certainty to particular writers.

"Wit" is taken as synonymous with genius and creative imagination, and as standing in contrast with pure intellect. Similarly "taste" refers to the intuitive æsthetic judgment

<sup>1</sup> See my article on *Polygonal Forms* for a discussion of the "golden section."

<sup>2</sup> See Croce, *History of Æsthetics* (second English edition, 1922), p. 185.

<sup>3</sup> For an account of this development see Croce, *loc. cit.*, Chap. III.

based on æsthetic feeling, which may, however, be in part susceptible of intellectual analysis.

The notion of "taste" is inherent in our analysis of the act of æsthetic perception: first the effort of attention, then the intuitive æsthetic judgment, dependent upon "taste," and finally analysis.

"Wit," or the faculty of creative imagination, is evidently closely allied with "taste," or the appreciative faculty. In fact wit may be regarded as taste transposed to a higher key.

## 9. DESCARTES

The rationalistic philosophy of Descartes gave to reasoning a position of first importance: *cogito ergo sum*. Himself a creative mathematician, he regarded mathematical reasoning as the model from which to start. The formalistic universe to which a mixture of speculation and reason led him, contained many metaphysical, physical, biological, physiological, and psychological doctrines of great originality and interest. Unfortunately, in his dualistic account of mind and matter, imagination was regarded as caused by the play of the animal spirits upon the mind. In consequence, poetry and other works of art were tolerated only in so far as they were in accord with reason. It is very significant of this general attitude that Descartes scarcely refers to æsthetic questions in his extensive writings.

## 10. LEIBNITZ

The formalistic universe of Leibnitz, also a great mathematician, provided for every conceivable type of being, each having its representative monad. Thus he was able to admit æsthetic facts without any difficulty. Æsthetic perceptions and judgments appear as valid forms of knowledge. Such knowledge is as *clear* as intellectual knowledge, but

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differs from the latter in that it is *confused* instead of *distinct*.<sup>1</sup> Here he follows a classification employed earlier by Descartes.

Leibnitz's celebrated enigmatic definition of music as "counting performed by the mind without knowing it is counting" will be seen to be consonant with our thesis that the density of certain orderly relations among the notes, estimated intuitively, measures the æsthetic effect. This definition of Leibnitz may also be regarded as partly substantiated by other musical researches to which we shall refer.

### 11. BOILEAU. DE CROUSAZ

Boileau and other writers of the Cartesian school soon tried to extend Cartesian doctrines to the æsthetic realm. This attempt led inevitably to a completely intellectualized point of view. De Crousaz says in his *Traité du beau* of 1724: "Good taste makes us appreciate at first by feeling that which reason would have approved . . ." But this claim is certainly exaggerated in all of those cases where important connotative elements enter, as we have pointed out.

The main factors of the beautiful were considered by de Crousaz to be variety, unity, regularity, order, and proportion. Evidently these are essentially the same mathematical elements of order specified by Plato and Aristotle.

### 12. Vico

Vico, a jurist known generally for his "philosophy of history," in common with many others of his period, reacted strongly against the intellectualistic view of poetic art. He

<sup>1</sup> *De cognitione veritate et ideis* (1684).

considered imagination as embodied in poetry as of equal importance with intellect as embodied in metaphysics, and independent of it. Furthermore he identified poetry with language, thus emphasizing first of all the expressive power of art.<sup>1</sup> For this reason Croce, himself of the same school, regards Vico as "the real revolutionary who by putting aside the concept of probability and conceiving imagination in a novel manner actually discovered the true nature of poetry and art and, so to speak, invented the science of *Æsthetics* . . ."<sup>2</sup>

However, after the acceptance of such a vigorous affirmation of the expressiveness of art, it remains (in our opinion) to discover how art is made expressive. This analytic phase was repellent to Vico. To us on the contrary it appears to be the chief part of the science of *æsthetics*.

### 13. RAMEAU

The musician Rameau in his *Traité de l'harmonie* of 1737 penetrated deeply into the nature of harmony, and thus contributed to the understanding of certain *æsthetic* effects in music. He observed that a musical note is in general composite, being composed of a pure fundamental note and overtones which can be heard, and that notes differing by an octave are so similar in their *æsthetic* effect as to be almost identical. These facts lead directly, as we shall see, to an understanding of the Western scale. They also lead to the notion of the fundamental bass or root of a chord, due to Rameau, and explain why this bass must in general proceed by a fourth or fifth, up or down to its harmonically nearest notes.

The cogency of his development seemed to Rameau so

<sup>1</sup> *Scienza nuova prima* (1725).

<sup>2</sup> *Loc. cit.*

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complete that he entitled a later work (1750) *Démonstration du principe de l'harmonie*. However the acute mathematician, d'Alembert, gave a clear presentation of Rameau's work in which he stated the requisite empirical rules without attempt at their demonstration.<sup>1</sup>

Rameau's treatment of harmony and other later treatments of harmony differ from that which we shall propose in two fundamental respects: they are qualitative rather than quantitative, since they aim only to exclude inadmissible chords and chordal sequences; and empirical exceptions are introduced wherever necessary, whereas our theory proceeds uniformly on the basis of the specific evaluation of simple æsthetic factors.

### 14. EULER

In his *Tentamen novæ theoriæ Musicæ* of 1739, the mathematician Euler developed a theory of consonance based upon the Pythagorean law. This was interpreted in the general sense that the smaller the integers expressing the vibration ratio of two notes, the more consonant the notes involved will be. In this way he is led to a simple empirical rule for estimating the degree of harmoniousness of any musical interval or chord, which in general corresponds with the observed facts.

The degree of harmoniousness is of course entirely distinct from that of agreeableness or æsthetic measure. For instance, unison and the octave are the most harmonious of all intervals, but are not the most agreeable. Nevertheless it is extremely interesting that Euler should have formulated a quantitative rule for the measurement of harmoniousness.

Euler's general concept of the nature of æsthetic enjoyment was in entire agreement with our own as may be

<sup>1</sup> *Éléments de Musique, suivant les principes de M. Rameau* (1762).

gathered from the following general account of it as given by Helmholtz:<sup>1</sup> "The more easily we perceive the order which characterizes the objects contemplated, the more simple and perfect they will appear, and the more easily and joyfully shall we acknowledge them. But an order which costs trouble to discover, although it will indeed also please us, will associate with that pleasure a certain degree of weariness and sadness."

#### 15. HOGARTH

The artist Hogarth in his *Analysis of Beauty* (1753) attempted an analysis of the æsthetic factors in painting. The æsthetic problem here is vague and difficult. He did no more than enumerate formal factors such as symmetry, variety, uniformity, simplicity, intricacy, quantity, and the factor of convincing representation. He ascribed an especial beauty to a serpentine line which he called the Line of Beauty.

#### 16. BURKE

At about the same time the statesman and philosopher Edmund Burke in his *Enquiry into the Origin of our Ideas of the Sublime and the Beautiful* (1756) separates the sensuous and imaginative factors in the works of art. These latter are conceived of as essentially mimetic. Furthermore he endeavored to classify the various æsthetic factors upon which depend the beauty of an object: smoothness, variety in the parts, lack of angularity, etc.

#### 17. HEMSTERHUIS

The philosopher Hemsterhuis in a *Lettre sur la sculpture* published in 1769 gave a definition of the beautiful which has become very well known: "the beautiful is that which

<sup>1</sup> *Tonempfindungen* (1862).

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gives the greatest number of ideas in the shortest space of time."

This definition contains much of the essence of our fundamental formula taken in a qualitative sense. For, the ideas to which he refers correspond to our connotative elements of order in the æsthetic object.

### 18. KANT

The great philosopher, Kant, devoted much attention to æsthetic questions. To him, æsthetic ideas expressed in works of art are supplementary to logical ideas or concepts and reinforce them: such works "make us think more than we can express in a given concept by means of words and give us an æsthetic idea which serves to this rational idea instead of a logical representation."<sup>1</sup>

Evidently this doctrine emphasizes the expressive character of art, and points toward the associations and feelings aroused by the contemplation of a work of art as of vital importance.

Kant expressed clearly the distinction between sensuous, emotional, moral, or intellectual feeling and æsthetic feeling to which we alluded at the beginning; sensuous and emotional feeling is excluded because the beautiful must please "without interest"; moral feeling is excluded because it must please "without the representation of an end"; intellectual feeling because it must please "without concepts."

Throughout Kant's writings there is evident a strong tendency toward the mystical view towards art. There is little which can be regarded as analytical.

### 19. SCHILLER. HEGEL

The followers of Kant and his metaphysical idealism continued in the same nebulous speculative realm of thought.

<sup>1</sup> *Kritik der Urtheilskraft* (1790).



Their æsthetic writings culminate in striking mystic phrases such as: beauty is "living form" (Schiller); "art cancels matter through form" (Schiller); beauty is the "sensible appearance of the Idea" (Hegel).

## 20. HERBART

Among those of the Kantian school, Herbart seems to us the most suggestive in his point of view towards æsthetics, despite a certain dry formalism.

For Herbart beauty consists primarily in relations, that is, in elements of order according to our terminology. Furthermore, art is two-sided: it possesses content, which is not properly æsthetic, and form, which is of its essence.<sup>1</sup> Evidently his division into form and content corresponds closely with our own into formal and connotative elements of order.

## 21. SCHLEIERMACHER

The theologian and philosopher Schleiermacher insisted on the inspired expressional nature of art, as so many of the Kantian school and others have done. What especially interests us is his realization that a work of art must be compared with others of the same kind. In his insistence upon this fact he is very explicit: "there is no difference in works of art except in so far as they can be compared in respect of artistic perfection." "In this respect the biggest, most complicated canvas is on a level with the smallest arabesque, the longest poem with the shortest . . . This proposition must be adhered to absolutely, if irrelevant elements are not to enter everywhere."<sup>2</sup>

By this insistence Schleiermacher clarified the funda-

<sup>1</sup> *Einleitung in die Philosophie* (1813).

<sup>2</sup> *Vorlesungen über Aesthetik* (published in 1842 after his death).

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mental problem of æsthetics. In the various applications of the mathematical theory it is essential that the class of æsthetic objects be closely prescribed. We are entirely in agreement with Schleiermacher in this respect.

### 22. POE

The American poet, Poe, always a theorist and many times an extravagant one, was led by his study of poetry to an æsthetic theory in which is formulated, apparently for the first time, the conjecture that æsthetic elements of order have a definite weight. The following quotation from his essay *The Rationale of Verse* (1843) embodies his conjecture:

“Let us examine a crystal. We are at once interested by an equality between the sides and between the angles of one of its faces: the equality of the sides pleases us; that of the angles doubles the pleasure. On bringing to view a second face in all respects similar to the first, this pleasure seems to be squared; on bringing to view a third, it appears to be cubed, and so on. I have no doubt indeed, that the delight experienced, if measurable, would be found to have exactly mathematical relations such as I suggest; that is to say, as far as a certain point, beyond which there would be a decrease in similar relations.”

So far as I know, this is the only affirmation of the kind to be found in earlier æsthetic theories.

In the application of this general idea to poetry Poe dealt with the formal elements of rhyme (interpreted to include alliteration and assonance) and metre. For rhyme and metre were respectively the “equality” of sound and the “equality” of time, both appreciable by the ear. He formulated no precise definition of the corresponding musical quality in poetry, but his poems are remarkable in this respect.

## 23. SPENCER

The positivist philosopher Spencer made several penetrating remarks in the field of æsthetics. In his *Philosophy of Style* (1852) he asserts that the effective cause of style is economy of effort. The notion of economy of effort, independently developed later by Fechner, is evidently in accord with our theory. In the nearly contemporaneous *Origin of Architectural Styles* Spencer ascribes the beauty of architectural form to uniformity and symmetry, qualities exhibited in the forms of nature; in other words, the æsthetic effectiveness of symmetry in architecture for instance is due in part to its association with the symmetry of the human body and other natural forms. Moreover in his *Origin and Function of Music* (1857), he advances the view that music is derivative from language. In our opinion the connotative element in music, produced by this linguistic origin, is beyond the reach of any analytic theory such as that here advanced.

## 24. HELMHOLTZ

Helmholtz, physiologist, physicist and mathematician, undertook the systematic examination of the physical and physiological basis of sensations of tone. The *Tonempfindungen* (1862) constitutes a veritable *principia* of the subject. In particular the existence of summation and difference tones is established, and explained upon a mathematical basis.

His genetic account (following Rameau) of chords and the Western scale is complete, in that these are shown to arise naturally. In other words the associative structure involved is traced in all of its ramifications.

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Helmholtz only considers briefly the more complicated question of chordal sequences and in this respect does not go much beyond Rameau.

In his work there is to be found no indication of any quantitative outlook. However he clearly adheres to the general analytic point of view, for he says: "No doubt is now entertained that beauty is subject to laws and rules dependent on the nature of human intelligence" which "are not consciously present to the mind, either of the artist . . . or the observer." Indeed "it is an essential condition that the whole extent of the regularity and design of a work of art should not be apprehended consciously. It is precisely from that part of its regular subjection to reason which escapes our conscious apprehension that a work of art exalts and delights us."

Thus Helmholtz believes that art depends upon definite laws which may be discovered. He denies, however, that art can satisfy after its structural laws are understood.

It seems to us that this last conclusion is not fully justified. In fact it is in an involved *density* of elements of order, obvious or more or less concealed, that we shall find the secret of successful musical form. These types of order are so varied and numerous, however, that the same æsthetic effect is felt regardless of the possibility of systematic enumeration which reveals the constituent elements involved.

### 25. SYLVESTER

The mathematician and casual poet, Sylvester, undertook in his *Laws of Verse* published in 1870 to reduce versification to definite principles. It is evident that he was influenced by the earlier work of Poe in this direction, with whom he finds himself in general agreement.

Although Sylvester dealt mainly with rhythm, his contribution is to be regarded as a distinct advance in the æsthetics of verse.

Sylvester divides the technical (formal) side of verse which he calls the "Rhythmic" into three branches: "Metric," about which he accepts Poe's doctrines; "Chromatic" dealing with the tonal side into which he does not enter, and the "Synectic" dealing with the "continuous aspect of the Art." In Synectic, the central concept is that of "Phonetic Syzygy," "to which we must attend in order to secure that coherence, compactness, and ring of true metal, without which no versification deserves the name of poetry."

This concept of Phonetic Syzygy will be seen later to correspond roughly to our æsthetic measure of musical quality in poetry.

## 26. HANSLICK

The philosopher Hanslick has exerted a strong influence against the opinion that music is beautiful primarily because of its mimetic linguistic power rather than as the embodiment of abstract form.<sup>1</sup>

In speaking of the beautiful in music he says: "*Its nature is specifically musical.* By this we mean that its beauty is not contingent upon, or in need of any subject introduced from without, but that it consists wholly of sounds artistically combined. The ingenious coordination of intrinsically pleasing sounds, their consonance and contrast, their flight and re-approach, their increasing and decreasing strength—this it is which in free and unimpeded forms presents itself to our mental vision."

<sup>1</sup> *Von musikalische Schönen* (1874), also *The Beautiful in Music* (translation, 1891).

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"It is extremely difficult to define this self-subsistent and specifically musical beauty."

"There is no art which, like music, uses up so quickly such a variety of forms."

"Mathematics though furnishing an indispensable aid to the study of the physical aspect of music must not be over-rated . . . No mathematical calculation ever extends into a composition."

Evidently Hanslick goes beyond Helmholtz in affirming that musical beauty is formal and is made up of elements of order which are complex but can be effectively analyzed. He perceives also that each type of music is limited by the allowance of means which it employs, as we shall assert. His reference to mathematics is evidently to its use in physical acoustics.

Our own theory is of course devoid of any mathematics except mere enumeration. Moreover, it is not supposed by us that the elements of order are counted when music is heard, but rather that they are intuitively felt in their aggregate weight.

### 27. FECHNER

With the rise of modern experimental psychology it was inevitable that the field of æsthetics should be approached in the light of the new ideas. The wellknown psychologist Fechner attempted to found a science of æsthetics "from below," of this kind.<sup>1</sup>

In the long list of "principles" and "laws" to which he is led, an important rôle is assigned to association. We have seen how important this rôle is.

Fechner contrasts the direct factor in æsthetic perception with this associative factor. For example, in the perception

<sup>1</sup> *Vorschule der Aesthetik* (1876).

of an orange, the direct factor would be essentially that of a yellow sphere; the indirect, that of a delicious tropical fruit.

Undoubtedly Fechner did not consider the symmetry of the orange to give rise to an association. Our reason for so regarding it may be stated as follows: all objects possessing such symmetry are associated by means of the uniform tactual and visual technique involved; in general this symmetry is desirable, so that the association has a positive tone of feeling.

Aside from his recognition of the importance of associations for æsthetic perception, Fechner's main contribution is methodological. He was the first to endeavor to treat the fundamental æsthetic problem for simple classes of æsthetic objects such as rectangles by direct experiment.

## 28. LIPPS

The psychologist Lipps stressed the importance of the empathetic factor in æsthetic perception, by which the self is identified with the artistic object.<sup>1</sup> Such identification is obviously important in the appreciation of a statue. Here the complete set of associations induced by the act of attentive perception necessarily leads to such an empathic response. The importance of empathy seems less clear in other cases.

## 29. GURNEY

The philosopher and psychologist Gurney in his *Power of Sound* (1889) undertakes to appraise the rôle of the formal elements of order in music. Unfortunately he overlooks the presence of any but the most obvious formal elements and does not perceive in the least how such order should be

<sup>1</sup>*Ästhetischen Faktoren der Raumschauung* (1891).

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measured. In consequence he is forced to the conclusion that music is "Ideal Motion" and that "the essential characteristic of the complete Ideal Motion is an absolutely unique beauty perceived by an absolutely unique faculty . . ." Of course such a conclusion is entirely mystical.

A specific solution of the problem of melody, which Gurney thus declares to be insoluble, will be tentatively proposed by us later.

### 30. CROCE

The æsthetician and philosopher Croce follows in the steps of Vico and others by insisting that art is expressive: "Art is expression of impressions . . ."<sup>1</sup>; art is "lyrical intuition."<sup>2</sup> For him knowledge is divided into intuitive knowledge and conceptual knowledge; the first finds its expression in art, the second in science and philosophy.

Such general philosophical definitions and classifications, however true, can never serve as the point of departure for a science of æsthetics. They are self-limited, and form a kind of philosophic citadel from which any and all more definite conclusions can be conveniently assailed.

### 31. LANIER

The American poet Lanier has written an important book, *The Science of English Verse* (1901), in which his starting point is the definite parallelism between poetry and music: "Perhaps no one will find difficulty in accepting the assertion that when formal poetry or verse . . . is repeated aloud, it impresses itself upon the ear as verse only by means of certain relations existing among its component words considered purely as sounds, without reference to their

<sup>1</sup> *Loc. cit.*

<sup>2</sup> *Æsthetics*, Encyclopedia Britannica, 14th edition (1929).



associated ideas"; "the sound-relations which constitute music are the same as those which constitute verse, and the main distinction between music and verse is, when stated with scientific precision, the difference between the scale of tones used in music and the scale of tones used by the human speaking-voice."

In developing this thesis, Lanier confines his attention mainly to the phenomenon of rhythm, where his views differ substantially from those of Poe.

There is of course no effort at a quantitative consideration of rhythm by Lanier.

### 32. ROSS. POPE

My colleague, Dr. Denman W. Ross, has systematically treated design and painting by analysis of the formal æsthetic factors which enter.

In his *Theory of Pure Design* (1907) he says: "The Beautiful is revealed, always, so far as I know, in the forms of Order, in the modes of Harmony, of Balance, and Rhythm. While there are many instances of Harmony, Balance, and Rhythm which are not particularly beautiful, there is, I believe, nothing really beautiful which is not orderly in one or the other, in two, or in all three of these modes. In seeking the Beautiful, therefore, we look for it in instances of Order, in instances of Harmony, Balance, and Rhythm. We shall find it in what may be called supreme instances."

Thus Ross defines that to be beautiful which is a "supreme instance of Order."<sup>1</sup> This concept, although entirely qualitative, is evidently akin to that embodied in our general theory.

Ross has classified the formal elements of order in paint-

<sup>1</sup>*On Drawing and Painting* (1912).

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ing and design as due to repetition, sequence, and balance as applied to tones, sizes, and shapes.

However, the conditions under which such order becomes "emotionally, as opposed to intellectually, appreciable order" are none too clear; the importance of this question has been pointed out by my colleague Professor Arthur Pope.<sup>2</sup>

### 33. THE EASTERN VIEW OF ART

So far as I can discover, the general analytic treatment of art, such as leads to æsthetics properly so called, is not to be found in the East. Instead there are charming literary anecdotes about artists and their work, as well as definite technical rules for the artist.

There are some slight indications of a more general point of view; for instance in Chinese art there are such pronouncements as the Six Canons of Painting of Hsieh Ho: (1) vitality, (2) anatomical structure, (3) conformity with nature, (4) suitability of coloring, (5) artistic composition, (6) finish. Likewise in Indian music there is much elaboration of the modes of musical scales.

### 34. CONCLUDING REMARKS

In the above account no attention has been given to various subjective theories such as are often formulated by poets and other creative artists because of their pragmatic usefulness. Of course the artist, on account of his highly developed powers of intuitive judgment, is not apt to stray far from the right track because of a wrong theory. In fact such a theory may prove valuable by suggesting novel combinations and experiments.

<sup>2</sup>*An Introduction to the Language of Drawing and Painting. I. The Painter's Terms* (1929).

This account of earlier æsthetic developments shows clearly that the principal æsthetic advances of the past can be conveniently described and interpreted in the light of our mathematical theory, at least in so far as they are concerned with the precise formal side rather than the elusive connotative side of the æsthetic experience.